Math 43
Quiz 7
Wed Nov 21, 2012 (DUE Mon Nov 26, 2012 @ START OF CLASS)

NAMES YOU ASKED TO BE CALLED IN CLASS: _____

SCORE: ___ / 35 POINTS

**SHOW ORGANIZED & PROPER WORK & SIMPLIFY YOUR FINAL ANSWER TO RECEIVE FULL CREDIT**

- Sign below to confirm that the work shown on this quiz is strictly your own work.
- You may have consulted your textbook, your notes and the class handouts, and used your calculator,
- but you did NOT consult other people, websites, software or other outside sources of help.

SIGNATURE: _____

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Find the center and radius of the sphere $x^2 + y^2 + z^2 - 8x + 10y - 6z + 2 = 0$. SCORE: ____ / 3 PTS

$x^2-8x+16 + y^2+10y+25 + z^2-6z+9 = -2+16+25+9$

$(x-4)^2 + (y+5)^2 + (z-3)^2 = 48$ (1) CENTER $(4,-5,3)$ (1) RADIUS $4\sqrt{3}$ (1)

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Consider the vector $\vec{m} = 3\vec{i} - 2\vec{j}$. SCORE: ____ / 4 PTS

[a] Find a unit vector perpendicular to $\vec{m}$.

$\langle 3,-2\rangle \cdot \langle a,b\rangle = 0$

$3a-2b=0$ (1) OR LET $a=2, b=3$

$\frac{1}{\|\langle 2,3\rangle\|}\langle 2,3\rangle = \frac{1}{\sqrt{13}}\langle 2,3\rangle = \left\langle \frac{2\sqrt{13}}{13}, \frac{3\sqrt{13}}{13} \right\rangle$ (1)

[b] Find the direction angle of $\vec{m}$. (Your answer should be in radians.)

$\|\vec{m}\| = \sqrt{13}$ (1/2)

$\cos\theta = \frac{3}{\sqrt{13}}$

$\sin\theta = \frac{-2}{\sqrt{13}}$ OR $\tan\theta = -\frac{2}{3}$

REFERENCE ANGLE $= 0.588$

$\theta = 2\pi - 0.588 = 5.695$ (1)

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Consider the vectors $\vec{f} = 3\vec{i} - 2\vec{k}$ and $\vec{g} = -4\vec{j} + \vec{k}$. SCORE: ____ / 9 PTS

[a] Find the angle between $\vec{f}$ and $\vec{g}$. (Your answer should be in radians.)

$\theta = \cos^{-1}\frac{\vec{f}\cdot\vec{g}}{\|\vec{f}\|\|\vec{g}\|} = \cos^{-1}\frac{3(0)+0(-4)+(-2)1}{\sqrt{13}\sqrt{17}} = \cos^{-1}\frac{-2}{\sqrt{221}} = 1.706$ (1) (1)

[b] Find a unit vector perpendicular to both $\vec{f}$ and $\vec{g}$.

$\vec{f}\times\vec{g} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -2 \\ 0 & -4 & 1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ 3 & 0 \\ 0 & -4 \end{matrix} = -12\vec{k} - 3\vec{j} - 8\vec{i}$

$= \langle -8,-3,-12\rangle$ (1/2)

$\frac{1}{\|\langle -8,-3,-12\rangle\|}\langle -8,-3,-12\rangle = \frac{1}{\sqrt{217}}\langle -8,-3,-12\rangle = \left\langle \frac{-8\sqrt{217}}{217}, \frac{-3\sqrt{217}}{217}, \frac{-12\sqrt{217}}{217} \right\rangle$ (1)

[c] Write $\vec{f}$ as the sum of 2 vectors, one parallel to $\vec{g}$ and one perpendicular to $\vec{g}$.

$\text{PROJ}_{\vec{g}}\vec{f} = \frac{\vec{f}\cdot\vec{g}}{\vec{g}\cdot\vec{g}}\vec{g} = \frac{-2}{17}\langle 0,-4,1\rangle = \langle 0, \frac{8}{17}, \frac{-2}{17}\rangle$ (1) (1/2)

$\langle 3,0,-2\rangle - \langle 0, \frac{8}{17}, \frac{-2}{17}\rangle = \langle 3, -\frac{8}{17}, -\frac{32}{17}\rangle$ (1)

$\vec{f} = \langle 0, \frac{8}{17}, \frac{-2}{17}\rangle + \langle 3, -\frac{8}{17}, -\frac{32}{17}\rangle$ (1)

Let $P$ be the point $(-2,-4,1)$. Let $Q$ be the point $(3,-2,-1)$. Let $R$ be the point $(-1,-3,-2)$. **SCORE: ____ / 19 PTS**

Let $\vec{u}$ be the vector with initial point $Q$ and terminal point $P$.

Let $\vec{w}$ be the vector with initial point $R$ and terminal point $P$.

[a] What octant is $R$ in?

QUADRANT 3 + 4 = OCTANT 7 (1)

[b] Write $\vec{u}$ in component form.

$\langle -2-3, -4--2, 1--1\rangle = \langle -5,-2,2\rangle$ (1/2)

[c] Write $\vec{w}$ as a linear combination of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

$\langle -2--1, -4--3, 1--2\rangle = \langle -1,-1,3\rangle = -\vec{i}-\vec{j}+3\vec{k}$ (1/2)

[d] Find the magnitude of $\vec{w}$.

$\sqrt{1+1+9} = \sqrt{11}$ (1)

★ MUST BE IN $\vec{i},\vec{j},\vec{k}$ FORM
NOT $\langle , , \rangle$ FORM

[e] Find a unit vector in the opposite direction as $\vec{u}$.

$-\frac{1}{\|\langle -5,-2,2\rangle\|}\langle -5,-2,2\rangle = -\frac{1}{\sqrt{33}}\langle -5,-2,2\rangle = \left\langle \frac{5\sqrt{33}}{33}, \frac{2\sqrt{33}}{33}, \frac{-2\sqrt{33}}{33}\right\rangle$ (1) (1) (1)

[f] Find a vector of magnitude 4 in the same direction as $\vec{w}$.

$4\cdot\frac{1}{\sqrt{11}}\langle -1,-1,3\rangle = \left\langle \frac{-4\sqrt{11}}{11}, \frac{-4\sqrt{11}}{11}, \frac{12\sqrt{11}}{11}\right\rangle$ (1) (1) (1/2)

[g] Find the area of triangle $PQR$.

$$\vec{U}\times\vec{W} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -2 & 2 \\ -1 & -1 & 3 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -5 & -2 \\ -1 & -1 \end{matrix} = \begin{matrix} -6\vec{i} - 2\vec{j} + 5\vec{k} \\ +2\vec{i} + 15\vec{j} - 2\vec{k} \end{matrix} = \langle -4,13,3\rangle$$ (1/2)

$\frac{1}{2}\|\langle -4,13,3\rangle\| = \frac{1}{2}\sqrt{194}$ (1)

[h] Find the general equation of the plane passing through $P$, $Q$ and $R$.

NORMAL VECTOR = $\vec{U}\times\vec{W}$

$-4(x--2)+13(y--4)+3(z-1)=0$

(1) $-4x+13y+3z = -41$ (1)

[i] Find parametric equations for the line which passes through $R$ and is also parallel to $\vec{u}$.

$$\begin{aligned} x &= -1-5t \\ y &= -3-2t \\ z &= -2+2t \end{aligned}$$ (1/2)

[j] Find symmetric equations for the line which passes through $Q$ and is also perpendicular to the plane $-x+5y-7z=9$.

DIRECTION VECTOR = $\langle -1,5,-7\rangle$

$\frac{x-3}{-1} = \frac{y+2}{5} = \frac{z+1}{-7}$ (1/2)